Errata

Nicholson, *Elementary Linear Algebra*, 1st Edition

Each of the following points represents how the text should *appear, and are listed by page number as well as by question, example, line number, or other indicator.*

Page 71
Example 2:

Example 2. *Given elementary matrices*

$$E_1 = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

write down the inverses.

Page 84
Exercises, Question #1:

1. In each case use the given LU-factorization of A to solve the system $AX = B$ by solving $LY = B$ and $UX = Y$ by backward and forward substitution respectively.

Page 86
Question #12(b):

(b) Show that the factorization in (a) is unique if A is invertible.

Page 118
Question #16(b):

(b) $\begin{bmatrix} A & 0 & 0 \\ X & B & 0 \\ Z & Y & C \end{bmatrix}$

Page 120
Lines 28-31

Using this we can compute A^3 as follows:

$$A^3 = AA^2 = (PDP^{-1})(PD^2P^{-1}) = PD^3P^{-1}.$$

Continuing in this way (and noting that this works even if D is not diagonal), we obtain

Page 137
Footnote #8

[8]Suppose λ_1 and λ_2 are both dominant (possibly equal), that is $|\lambda_1| = |\lambda_2| > |\lambda_i|$ for each $i \geq 3$. Then $V_k \approx \lambda_1^k b_1X_1 + \lambda_1^k b_2X_2$ for sufficiently large k. A similar analysis applies if λ_1, λ_2 and λ_3 are all dominant.

Page 146
Question #5(f):

(f) From that point on, the hawk survival rate drops to 0.50. The biologists are alarmed that this may wipe out the hawk population. Test to see what the model is predicting, and explain what is happening.

Page 161
Question #20:

20. Express each of the following in polar form.

(a) $2 - 2i$	(b) $-2i$
(c) $-\sqrt{3} + i$	(d) $-3 - \sqrt{3}\, i$
(e) $5i$	(f) $5(1 + i)$

Page 167
Question #14:

[*Questions 14(c) and 14(d) have been removed.*]

Page 205
3.3.1 Position Vectors, Line 3

$$\vec{p} = \overrightarrow{OP} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [x \;\; y \;\; z]^T .$$

Page 206
Line 6

the position vector $\vec{p} = [x \;\; y \;\; z]^T$ of P, and read off its *components* x, y, and

Page 246
Question #16(d):

(d) Y-Expansion by 2, followed by the rotation through $\pi/2$.

Page 261
Independence Test, Step 2.

Step 2. *Show that the only way this can happen is the trivial one with all coefficients* $t_i = 0$.

Page 276
Exercises, Question #3(c)

(c) $\{[-4 \;\; 3 \;\; -2]^T, [-1 \;\; 1 \;\; 11]^T, [-3 \;\; 2 \;\; -9]^T\}$ in $\mathbb{R}^3$.

Page 278
Question #15(a):

15. (a) Find a basis of vectors in $\mathbb{R}^3$ consisting of vectors whose components sum to 1.

Page 279
Question #25:

25. Let U and W be any two subspaces of $\mathbb{R}^n$, and consider the subspaces

$U \cap W = \{X \text{ in } \mathbb{R}^n \mid X \text{ is in both } U \text{ and } W\}$

$U + W = \{X \text{ in } \mathbb{R}^n \mid X \text{ is a sum of a vector in } U \text{ and a vector in } W\}$

As in Exercise 23 §4.1. Show that $dim(U + W) = dim U + dim W - dim(U \cap W)$.
[*Hint*: Let $\{X_1, \dots, X_d\}$ be a basis of $U \cap W$, and by Theorem 3, extend it to bases $\{X_1, \dots, X_d, Y_1, \dots, Y_k\}$ and $\{X_1, \dots, X_d, Z_1, \dots, Z_m\}$ of U and W respectively. Show that $\{X_1, \dots, X_d, Y1, \dots, Y_k, Z_1, \dots, Z_m\}$ is a basis of $U + W$.]

Page 284
Theorem 3, Point (4)

(4) $A^T A$ *is an invertible* $n \times n$ *matrix.*

Page 285
Lines 1-3

that $X = 0$. So assume that $(A^T A)X = 0$. Write $AX = [y_1\ y_2\ \dots\ y_m]^T$ and compute

$$y_1^2 + y_2^2 + \cdots + y_m^2 = (AX)^T(AX) = (X^T A^T)AX = X^T(A^T AX) = X^T 0 = 0.$$

Page 285
Theorem 3, Point (4)

(4) AA^T *is an invertible* $m \times m$ *matrix.*

Page 288
Question #3(a)-(c):

(a) $rowA = span\{[1\ \ 0\ \ 0\ \ 1], [0\ \ 1\ \ 1\ \ 0], [0\ \ 0\ \ 0\ \ 1]\}$.
(b) $rowA = span\{1\ \ 2\ \ -1\ \ 3], [2\ \ -3\ \ 1\ \ 4], [4\ \ -13\ \ 5\ \ 6]\}$.
(c) $rowA = span\{[-1\ \ 0\ \ 3\ \ 2], [3\ \ 2\ \ 0\ \ -4], [9\ \ 8\ \ 9\ \ -10], [4\ \ 2\ \ -3\ \ -6]\}$.

Page 288
Question #4(c):

(c) $U = span\{[-1\ \ 0\ \ 3\ \ 2]^T, [3\ \ 2\ \ 0\ \ -4]^T, [9\ \ 8\ \ 9\ \ -10]^T, [4\ \ 2\ \ -3\ \ -6]^T\}$

Page 302
Solution, Line 1:

Solution. The columns of A are $C_1 = [1\ \ -1\ \ 0\ \ 0]^T$, $C_2 = [1\ \ 0\ \ 1\ \ 0]^T$ and

Page 321
Final line:

then use (c) and Theorem 4 §4.3.]

Page 334
Question #14, Line 3

$D = diag(\lambda_1, \cdots, \lambda_n)$ where $P^{-1} = P^T$ and each $\lambda_i \geq 0$, take $B =$

Page 350
Exercises, Question #1(c):

(c) Every symmetric matrix with positive determinant is positive definite.

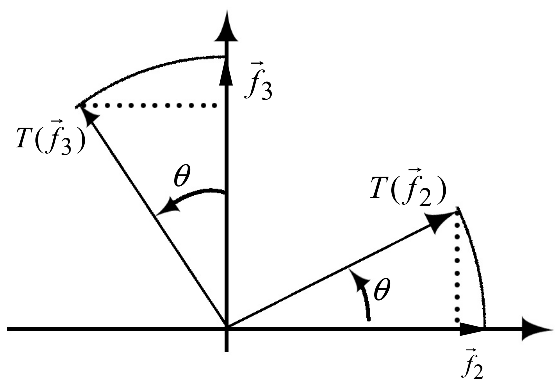
Page 352
Question #12:

12. If A is an $n \times n$ positive definite matrix, and if U is an $n \times m$ matrix of rank m, show that $U^T AU$ is positive definite [*Hint*: Theorem 3 §4.4.]

Page 358
7th-to-final line:

is uniquely determined *if it exists.* The natural temptation is to *define* T by

Page 365
Diagram #2:



Page 373
Question #6(b):

(b) Find $T([x_1\ x_2\ x_3\ x_4]^T)$ if $T: \mathbb{R}^4 \to \mathbb{R}^2$ is a linear transformation satisfying $T([1\ 1\ 0\ 0]^T) = [2\ -1]^T$, $T([0\ 1\ 0\ 0]^T) = [-2\ 3]^T$, $T([0\ 1\ 0\ 2]^T) = [1\ 0]^T$ and $T([0\ 0\ 1\ 0]^T) = [0\ -1]^T$.

Page 374
Question #17, Line 1:

Show that the composite of two distance preserving transformations of $\mathbb{R}^n$

Page 387
Question #24(a):

24. (a) Show that $det\, \overline{Z} = \overline{det\, Z}$ for every $n \times n$ complex matrix Z. [*Hint*: Use induction on n and apply the Laplace expansion.]

Page 397
Question #18(a):

18. (a) If X is an eigenvector of B corresponding to λ_i, show that $R(X) = \lambda_i$.

Page 412
Question #24(a):

24. (a) Show that $\mathbb{M}_{2,2} = \text{span}\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$.

Page 423
Question #31:

31. Show that $\mathbb{F}[0,1]$ is infinite dimensional.

Page 424
Question #35(c)-(d):

(c) If U and W are both finite dimensional, show that $U + W$ is also finite dimensional (even though V may not be finite dimensional). [*Hint*: If C and D are (finite) bases of U and W respectively show that $C \cup D = \{\mathbf{v} \text{ in } V \mid \mathbf{v} \text{ is in } C \text{ or } \mathbf{v} \text{ is in } D\}$ spans $U + W$.]

(d) If V is finite dimensional, show that $dim(U + W) = dim U + dim W - \dim(U \cap W)$. [*Hint*: if B is a basis of $U \cap W$, let (by Theorem 3) C and D be bases of U and W respectively each containing B. Show that $C \cup D$ is a basis of $U + W$.]

Page 453
Question #6(a):

(a) $V = \mathbb{P}_2$, $\mathbf{v} = x^2 - 3x + 4$, $B = \{x + 2, x^2 - 1, 1\}$.

Page454
Question #16:

16. Let $V \xrightarrow{T} W \xrightarrow{S} V$ be linear transformations such that $S \circ T = 1_V$. If $dim V = dim W$ is finite, show that also $T \circ S = 1_W$ (so T is an isomorphism and $S = T^{-1}$).

Page 466
Exercises, Question #1(h):

(h) If $\mathbf{v}$ is an eigenvector of T and $\mathbf{w}$ is in $ker T$, then $\mathbf{v} + \mathbf{w}$ is an eigenvector of T.

Page 482
Exercises, Question #1(f):

(f) If T is an isomorphism, the only T-invariant subspaces of V are V and $\{\mathbf{0}\}$.

Page 483
Question #8(b):

(b) Show that λ exists such that $T(\mathbf{v}) = \lambda\mathbf{v}$ for every vector $\mathbf{v}$ in V.

Page 483
Question #14:

14. Let $T : V \rightarrow V$ be a linear operator where $dim V = n$.
    (a) Show that $E_\lambda(T) \cap E_\mu(T) = \{0\}$ if $\lambda \neq \mu$.
    (b) If $\lambda \neq \mu$ are the only eigenvalues of an operator T, and if $dim(E_\lambda(T)) + \dim(E_\mu(T)) = n$, show that T is diagonalizable.
    (c) If $\lambda \neq \mu$ are the only eigenvalues of T and $dim V = 2$, show that T is diagonalizable.
    (d) Give an example where T has exactly two distinct eigenvalues but is not diagonalizable.

Page 495
Example 13, Line 6:

$$\mathbf{e}_m = \mathbf{b}_m - \frac{<\mathbf{b}_m, \mathbf{e}_0>}{\|\mathbf{e}_0\|^2}\mathbf{e}_0 - \frac{<\mathbf{b}_m, \mathbf{e}_1>}{\|\mathbf{e}_1\|^2}\mathbf{e}_1 - \cdots - \frac{<\mathbf{b}_{m-1}, \mathbf{e}_{m-1}>}{\|\mathbf{e}_{m-1}\|^2}\mathbf{e}_{m-1}.$$

Page 503
Question #19:

19. Let $W_1 \subseteq W_2 \subseteq \ldots$ be nonzero subspaces of an inner product space V such that $W_m \neq W_{m+1}$ for each m. Choose $\mathbf{b}_1 \neq \mathbf{0}$ in W_1 and, for $m \geq 2$, choose $\mathbf{b}_m$ in W_m but not in W_{m-1}. Show that $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \ldots\}$ is an infinite independent set.

Page 515
Question #8(a):

8. (a) Basic algebraic manipulations give

$$A = \frac{1}{2}\left(\begin{bmatrix} 1 & 4 \\ 0 & 7 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -3 & 4 \\ 0 & 8 \end{bmatrix}\right) = \begin{bmatrix} \frac{3}{2} & 2 \\ -\frac{3}{2} & \frac{11}{2} \\ -1 & \frac{11}{2} \end{bmatrix}.$$

Page 533
2.3 Diagonalization and Eigenvalues
Question #5, Line 3:

parameter family $t\begin{bmatrix}1\\0\end{bmatrix}$ if $c \neq 0$. So A is not diagonalizable by Theorem 5.

Page 538
Question #4:

4. These are very interesting recurrences where complex numbers are used to (diagonalize the matrix and) compute a recurrence of ordinary integers. For (b), we obtained the formula (with 2 decimals approximation)

$$V_k \approx (0.11)(-0.69)^k \begin{bmatrix}2.06\\-1.43\\1\end{bmatrix}$$
$$+(0.44+1.03i)(1.34+1.02i)^k \begin{bmatrix}0.09+0.33i\\0.46+0.35i\\1\end{bmatrix}$$
$$+(0.44-1.03i)(1.34-1.02i)^k \begin{bmatrix}0.09-0.33i\\0.46-0.35i\\1\end{bmatrix}$$

Page 550
Question #19:

19. For (a), $T(\vec{v})$ is already projected, so doing it again won't do anything. For (b), multiply the matrices from Theorem 1.
[ILAW: Exploration 3.5.4.]

Page 555
4.3 Dimension
Question #3(c):

(c) This set has 3 vectors in $\mathbb{R}^3$, and they are independent. Hence they form a basis of $\mathbb{R}^3$.

Page 559
4.5 Orthogonality
Question #6(b):

(b) The coefficients are $\frac{-4a+b+5c}{42}, \frac{-a+b-c}{3}, \frac{2a+3b+c}{14}$.

Page 561
Question #8(b):

(b) $U^{\perp} = span\{[5\ \ 3\ \ 1\ \ 0], [-5\ \ -2\ \ 0\ \ 1]\}$.

Page 563
Question #5(d):

$$(d)\ P = \frac{1}{\sqrt{182}} \begin{bmatrix} 3\sqrt{13} & 2\sqrt{14} & 3 \\ -\sqrt{13} & 0 & 13 \\ 2\sqrt{13} & -3\sqrt{14} & 2 \end{bmatrix}$$

Page 567
4.10 Complex Matrices, Question #2(c)

(c) $\|Z\| = 2\sqrt{5}$, $\|W\| = 2\sqrt{2}$; not orthogonal as $<Z, W> = 4 + 5i$ is nonzero.

Page 571
Question #4:

4. The zero vector is $[0, -1]$, and the negative of $[x\ \ y]$ is $[-x\ \ -2\ \ -y]$.
   Axiom S3 is verified as follows:

$$\begin{aligned} a \cdot [x\ \ y] \dotplus b \cdot [x\ \ y] &= [ax,\ ay + a - 1] \dotplus [bx,\ by + b - 1] \\ &= [ax + bx,\ (ay + a - 1) + (by + b - 1) + 1] \\ &= [(a + b)x,\ (a + b)y + (a + b) - 1] \\ &= (a + b) \cdot [x\ \ y]. \end{aligned}$$

Page 577
5.6 Invariant Subspaces, Question #2:

2. Show that $T^2 = T$ by showing that $T^2(\mathbf{b}_i) = T(\mathbf{b}_i)$ for each i. $B_0 = \{\mathbf{b}_1, \mathbf{b}_2 + 2\mathbf{b}_4\}$.

Page 578
Question #17:

17. We have $U = span\{\mathbf{v}, T(\mathbf{v}), T^2(\mathbf{v}), \ldots, T^{m-1}(\mathbf{v})\}$. If $\mathbf{w}$ is one of these generators of U, it is clear that $T(\mathbf{w})$ is in U, except if $\mathbf{w} = T^{m-1}(\mathbf{v})$. For this, use equation (**) preceding Theorem 6.

Page 580
Question #22(b):

(b) If $\|\ \ \|$ came from an inner product, there would (by Theorem 2) be a matrix P such that $[x\ \ y]\ P \begin{bmatrix} x \\ y \end{bmatrix} = |x| + |y|$ for all x and y. Show that this is impossible by examining various choices for x and y.