Preface

This textbook is a basic introduction to the ideas and techniques of linear algebra for first- or second-year students who have a working knowledge of high school algebra. Its aim is to achieve a balance among the computational skills, theory, and applications of linear algebra, while keeping the level suitable for beginning students. The contents are arranged to permit enough flexibility to allow the presentation of a traditional introduction to the subject, or to allow a more applied course. Calculus is not a prerequisite; places where it is mentioned are clearly marked and may be omitted.

Linear algebra has wide application to the mathematical and natural sciences, to engineering, to computer science, and (increasingly) to management and the social sciences. As a rule, students of linear algebra learn the subject by studying examples and solving problems. As in the third edition, more than 330 solved examples are included, many of a computational nature, together with a wide variety of exercises. In addition, a number of sections are devoted to applications and to the computational side of the subject. These are optional, but they are included at the end of the relevant chapters (rather than at the end of the book) to encourage students to browse.

The examples also play a role in motivating theorems, although most proofs are included at a level appropriate to the student. This means that the book can be used to give a course emphasizing computation and examples (and omitting many proofs) or to give a more rigorous treatment. A few longer proofs are deferred to the end of the chapter.

The **fourth edition**, like the third, maintains a balance between the abstract theory and matrix computations and applications. It contains two novel features. First, diagonalization is now treated in Chapter 3 (using only determinants and matrix inverses), thereby opening up a wealth of applications early in the book. The second innovation is the introduction of tough concepts such as independence, subspaces, spanning, dimension and linear transformations in $\mathbb{R}^n$ in a new "bridging" chapter (Chapter 5). This gives the students a chance to assimilate these ideas before the introduction of abstract vector spaces, and so mitigates the effect of "hitting the wall".

New Features in the Fourth Edition

- Diagonalization and eigenvalues are now introduced early (in Chapter 3 using only determinants and matrix inverses), and so can be presented to beginning students in their first course. This unique feature, suggested originally by the University of Calgary Engineering Faculty, opens up a number of important applications. For example, ecological models are an excellent motivation for the students because they can relate them to the extinction of species and see the relevance to the real world.

- Chapter 5 is a new “bridging chapter” in which concepts such as subspaces, spanning, independence, dimension and linear transformations are introduced into $\mathbb{R}^n$. This avoids “hitting the wall” for students who first see these ideas in the setting of an abstract vector space, and serves as a good stopping point for a first semester course at the second-year level. (It provides an early, rigorous treatment of rank and diagonalization.)
- The last section of Chapter 5 introduces linear transformations in $\mathbb{R}^n$, building on examples in $\mathbb{R}^3$ such as reflections and rotations. It requires only Chapters 2 and 4 (the material needed from Section 5.1 can be easily supplied directly), and provides a much needed geometrical view of matrix algebra.
- Increased flexibility for the instructor to choose different routes through the material (see the Section Dependency Diagram following the Chapter Summaries).
- Numerous computational exercises with no book answer have been revised, allowing them to be used again for assignments.
- The material on block multiplication (in Section 2.2) has been shortened to emphasize only those features that occur later in the book. This eliminates tedious examples that are time-consuming in class.
- The proof (in Section 2.3) of the fundamental characterizations of invertibility has been simplified and shortened, and does not require elementary matrices. This simplifies the teaching of beginning students.
- Section 2.5 on elementary matrices has been shortened by about 30 percent, which makes it easier to cover when needed later in the course.
- New applications of diagonalization to linear recurrences (in Section 3.5) and population growth (in Section 3.6) are now included. These are unique to this book because they occur early in the text (requiring only matrix inverses and determinants) and give examples of the use of linear methods that the students can understand.
- An application to chemical reactions has been included at the end of Chapter 1, adding to applications to network flows and electrical networks in Chapter 1 and economic models and Markov chains in Chapter 2. These give real-world examples early in the book, and so motivate students in other disciplines, many of whom do not yet really understand why they are studying linear algebra.
- The section on positive definite matrices and the Cholesky factorization has been completely rewritten; it is now both shorter and easier to follow. The section on QR-factorization has also been rewritten.
- The appendix on linear programming has been moved to the text specific website at www.mcgrawhill.ca/college/nicholson.

Other Features

- Presentation of techniques in examples, with emphasis on concrete computations and on the algorithmic nature of some techniques, allows students to master new skills readily. The text has more than 330 solved examples that cover the basic techniques, illustrate the central ideas, and are keyed to the exercises in the book.
- A wide variety of exercises, which start with routine computational problems and progress to more theoretical exercises, helps students develop skills in an appropriate, logically paced fashion.

- End-of-Chapter applications, showing how linear algebra clarifies and solves problems, provide relevance for students. These are placed at the end of the chapter containing the necessary technologies.
- Answers to even-numbered computational exercises and selected others enable students to check the accuracy of their computation immediately.

Ancilliary Materials

For Instructors:

- **Instructor's Solutions Manual** (print)
  - Contains answers or solutions to all the exercises found in the book.
- **Instructor Web Site** (password protected) at www.mcgrawhill.ca/college/nicholson:
  - ***Instructor's Solutions Manual*** (pdf download)
  - ***Test Bank*** (pdf download)—over 100 problems covering the entire course, suitable for exams, with complete solutions.
  - ***Chapter and Section Dependencies Chart***
  - ***Suggested Course Outlines***
  - ***Linear Programming*** Content
  - ***Web-based Labs/Testing*** (ILAW—Interactive Linear Algebra on the Web) offers online testing and online labs, algorithmically generated quiz questions, automated grading, and instructor grade book.

For Students:

- **Partial Student Solutions Manual** (print)
  - Contains detailed solutions to selected even-numbered exercises.
- **Student Web Site** located at www.mcgrawhill.ca/college/nicholson:
  - ***Web-based Student Tutorial*** (ILAW—Interactive Linear Algebra on the Web)
    - ***Lessons***—Animated, audio-enhanced, step-by-step lessons cover most topics in the course
    - ***Explorations***—Hands-on, interactive exercises reinforce understanding of concepts
    - ***Quizzes***—Algorithmically generated, self-testing quizzes are automatically graded and provide students with detailed feedback explaining why a submitted answer is incorrect
    - ***Computing Tools*** for standard computations with cut and paste capability
    - ***Message Board*** to communicate with other students, ask questions, and work through solutions.

Chapter Summaries

Chapter 1: A standard treatment of Gaussian elimination is given. The rank of a matrix is introduced via the row-echelon form. Applications to network flows, electrical networks, and chemical reactions are provided.

Chapter 2: Matrix operations (including transposition) are introduced, and matrix inverses are defined and studied (the use of elementary matrices is minimized). The relationship of matrix algebra to linear equations is emphasized, and block multiplication is introduced with emphasis on those cases needed later in the book. LU-factorization is introduced. Applications to economic models and Markov chains are given.

Chapter 3: The Laplace expansion is stated (proved later) and used to define determinants inductively and to deduce the usual rules. The product rule and the adjoint formula are proved. Then eigenvalues and eigenvectors are defined and the diagonalization algorithm is presented, leading to applications to linear recurrences and population growth. A section on polynomial interpolation is included.

Chapter 4: Vector operations are defined (motivated by examples), the dot product and projections are introduced and used to solve problems in $\mathbb{R}^2$ and $\mathbb{R}^3$. Coordinates are introduced and straight lines and planes are studied (using the cross product). An application to least squares approximation is given.

Chapter 5: This is a "bridging" chapter between what has gone before and the abstract theory. Subspaces, spanning, independence, and dimension are introduced in the context of $\mathbb{R}^n$, and used to rigorously complete the study of rank and diagonalization. Quite independently of this, linear transformations $\mathbb{R}^n \to \mathbb{R}^m$ are introduced (motivated by $\mathbb{R}^2$ and $\mathbb{R}^3$) and their relationship to matrix multiplication is clarified.

Chapter 6: Building on $\mathbb{R}^n$, the basic theory of finite dimensional vector spaces is developed emphasizing examples like matrices, polynomials, and functions. The pace is slow as this is the first acquaintance many students have had with an abstract system. Applications to polynomials and differential equations are presented.

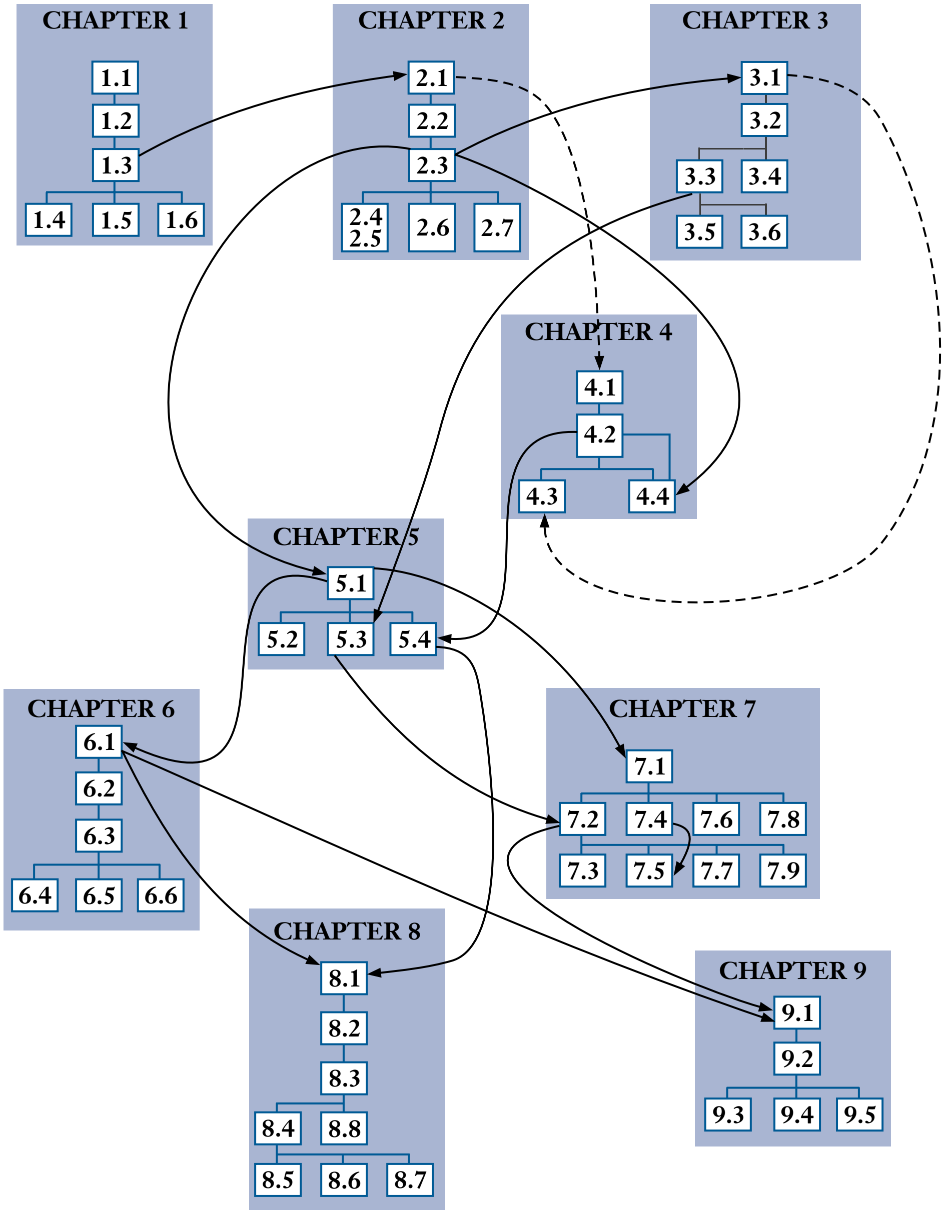
Chapter 7: Orthogonality is defined in $\mathbb{R}^n$ using the dot product, the Gram–Schmidt algorithm is given, and orthogonal projections are defined. These are used to study orthogonal diagonalization (obtaining the principal axis theorem), positive definite matrices (the Cholesky factorization), and QR-factorization. Three applications are given: quadratic forms, approximation and least squares, and systems of differential equations. A section on complex matrix algebra is included.

Chapter 8: General linear transformations are introduced, motivated by many examples from geometry, matrix theory, and calculus. The kernel and image are defined, the dimension theorem is proved, and isomorphisms are discussed. The matrix of a linear transformation is defined and the relationship between basis changes and similarity is revealed. Invariant subspaces are introduced and used to derive the block triangular form. An application to linear recurrences is given.

Chapter 9: General inner products are introduced; distance, norm, and the Schwartz inequality are discussed. The Gram–Schmidt algorithm is given, projections are defined, and the approximation theorem is proved (with an application to Fourier approximation). Finally, isometries are characterized.

Section Dependencies

The following chart suggests how the material introduced in each section draws on concepts covered in certain earlier sections. A solid arrow means that ready assimilation of ideas and techniques presented in the later sections depends on familiarity with the earlier chapter. A broken arrow indicates that some reference to the earlier section is made but the section need not be covered.



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