

The Quadratic Formula:

$$\text{If } y = ax^2 + bx + c, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example:

Use the quadratic formula to find the roots of each of the following.

a) $x^2 + x - 6 = 0$

b) $4x^2 - 12x + 9 = 0$

c) $2x^2 + 3x + 2 = 0$

Solution:

a) In this case, $a = 1$, $b = 1$, and $c = -6$. Therefore,

$$\begin{aligned} x &= \frac{-1 \pm \sqrt{1^2 - 4(1)(-6)}}{2(1)} \\ &= \frac{-1 \pm \sqrt{25}}{2} \\ &= \frac{-1 \pm 5}{2} \end{aligned}$$

$$\begin{aligned} x &= \frac{-1+5}{2} & \text{or} & & x &= \frac{-1-5}{2} \\ &= 2 & & & &= -3 \end{aligned}$$

The roots are 2 and -3.

b) In this case, $a = 4$, $b = -12$, and $c = 9$. Therefore,

$$\begin{aligned} x &= \frac{12 \pm \sqrt{12^2 - 4(4)(9)}}{2(4)} \\ &= \frac{12 \pm \sqrt{0}}{8} \\ &= \frac{3}{2} \end{aligned}$$

There is a double root at $\frac{3}{2}$.

c) In this case, $a = 2$, $b = 3$, and $c = 2$. Therefore,

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(2)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{-7}}{2}$$

$$x = \frac{-1 + \sqrt{-7}}{2} \quad \text{or} \quad x = \frac{-1 - \sqrt{-7}}{2}$$

The roots are not real.